8.812 Graduate Experimental Laboratory
MIT Department of Physics
Aug.2009 U.Becker

Exp. #2 Fermi constant from muon decay

Measure the lifetime of stopped cosmic muons. Determine /eliminate background events. Calculate the decay probability to extract the Fermi constant given the muon mass, or vice versa with error propagation. Roughly measure the electron momenta to determine that more than one neutrino is involved. 2a) Devise an experiment with different counters (flat).

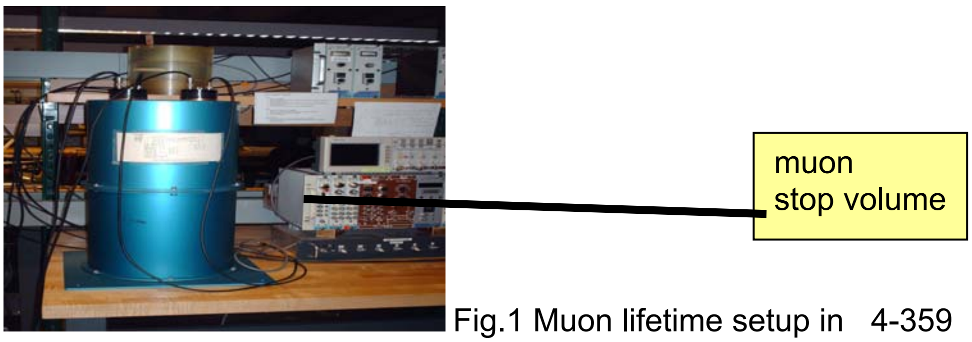


Fig.1 Muon lifetime setup in 4-359

In this experiment you will measure the decay curve and mean life of muons that have come to rest in a scintillator stop volume.

Preparation

2. Give the production mechanism for muons in the atmosphere. Assume primary protons with 1-50GeV impinge on nitrogen nuclei with cross section σ $A^{1/3}$ and density ~exp(height/8 km) Estimate roughly from which height they come. Why are muons particularly penetrating particles (as opposed to electrons or protons)? There are 1.25 times more μ^+ than μ^- -explain this observation. Physics COSMIC RAYS
3. A singly charged particle traveling in matter at nearly the velocity of light loses energy by Coulomb interactions with approximately 2 MeV/(gm/cm^2. (Bethe Bloch). How much energy is lost by a relativistic particle traversing the entire atmosphere?
4. Give reasons why muon decay must involve two different neutrinos.
5. From weak interaction theory obtain the absolute lifetime of a muon at rest.
6. Derive the energy spectrum of the electrons.
7. The weight of the cylinder of plastic scintillator used in the measurement of the muon mean life is 20.3 kg. Predict the rate of muon decay events in the cylinder. (See Appendix A).

Much of the material in this section is taken from the classic works by Bruno Rossi, [3–5]. Interstellar space is populated with extremely rarefied neutral and ionized gas (≈ 10^{-3} to 10^{3} atoms cm^{-2}), dust (≈ 1-10% of gas), photons, neutrinos, and high-energy charged particles consisting of electrons and bare nuclei of the elements with energies per particle ranging up to 10^{21} eV. The latter, called cosmic rays, constitute a relativistic gas that pervades the galaxy and significantly affects its chemical and physical evolution. The elemental composition of cosmic-ray nuclei resembles that of the sun, but with certain peculiarities that are clues to their origins. Most cosmic rays are generated in our galaxy, primarily in supernova explosions, and are confined to the galaxy by a pervasive galactic magnetic field of several microgauss. It is an interesting and significant fact that the average energy densities of cosmic rays, the interstellar magnetic

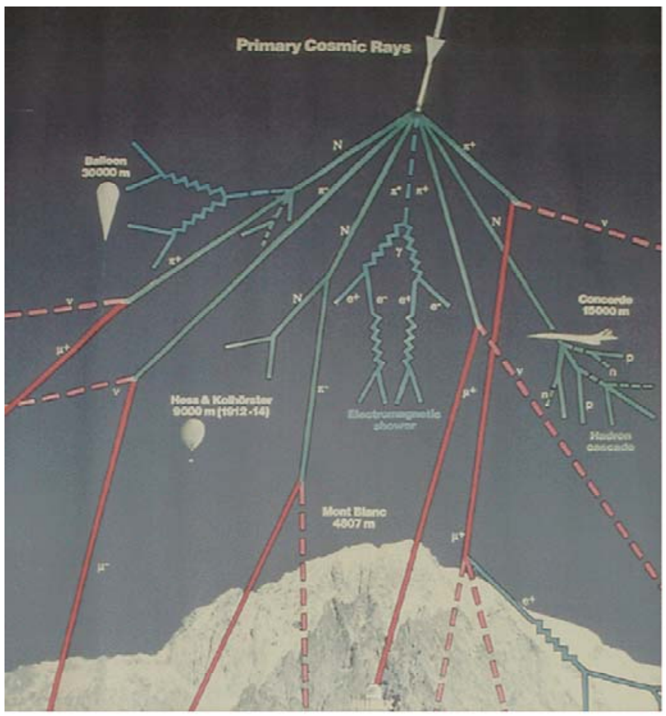


FIG. 1: (a) Production and decay of pions and muons in a representative high energy interaction of a cosmic-ray proton with a neutron in the nucleus of an air atom. (b) Masses and lifetimes of pions and muons.

When a primary cosmic ray (90% of which are protons, 9% helium nuclei, 1% other) impinges on the earth’s atmosphere it interacts with an air nucleus, generally above an altitude of 15 km. Such an interaction initiates a cascade of high-energy nuclear and electromagnetic interactions that produce an “air shower” of energetic particles spreading outward in a cylindrically symmetric pattern around a dense core. See Figure 1. As the shower propagates downward through the atmosphere the energy of the incident and secondary hadrons (nucleons, antinucleons, pions, kaons, etc.) is gradually transferred to leptons (weakly interacting muons, electrons and neutrinos) and gamma rays (high-energy photons) so that at sea level the latter are the principal components of “secondary” cosmic rays. Typical events in such a cascade are represented by the reactions shown in Figure 1. High altitude observations show that most of the muons that arrive at sea level are created above 15 km.

At the speed of light their trip takes ≈ 50 μsec.

In an ironic twist of history, these particles were believed to be Yukawa type (pions) until 1947 when they were found to be muons from $\pi^+ \rightarrow \mu^+ + \nu_\mu$ by Powell.

To check the reasonableness of the arrival rates of single pulses by measuring the size of the scintillator and estimate the total rate of muons R traversing it. You can use the following empirical formula that provides a good fit to measurements of the intensity of penetrating particles at sea level as a function of the zenith angle:

where $I_v = 0.83 \times 10^{-2}\ cm^{-2}\ s^{-1}\ str^{-1}$, and φ is the zenith angle (Rossi 1948). I(φ)dΩdAdt represents the number of particles incident upon an element of area dA during the time dt within the element of solid angle dΩ from the direction perpendicular to dA. By integrating this function over the appropriate solid angle you can estimate the expected counting rates of the detectors due to the total flux of penetrating particles from all directions, and the expected rate of coincident counts due to particles that arrive within the restricted solid angle defined by the telescope (See Appendix B). The rates of single events and coincidences for τ_μ are very important calculations and you should not proceed until you have determined there values!

MEASUREMENT OF THE MEAN LIFE OF MUONS AT REST

Muons were the first elementary particles to be found unstable, i.e. subject to decay into other particles. At the time of Rossi's pioneering experiments on muon decay the only other "fundamental" particles known were photons, electrons and their antiparticles (positrons), protons, neutrons, and neutrinos. Since then dozens of particles and antiparticles have been discovered, and most of them are unstable. In fact, of all the particles that have been observed as isolated entities the only ones that live longer than muons are photons, electrons, protons, neutrons, neutrinos and their antiparticles.

$n \rightarrow p e \nu_e$ Similarly, muons decay through the process $\mu^- \rightarrow e \nu_e \nu_\mu$

FIG. 3: Feynman diagrams of the muon decay process

During the period from 1940 to 1950 observations of muons stopped in cloud chambers and nuclear emulsions demonstrated that the muon decays into an electron and that the energy of the resulting electron may have any value from zero to approximately half the rest energy of the muon, i.e. ≈ 50 MeV. From this it was concluded that in addition to an electron the decay products must include at least two other particles, both neutral and of very small or zero rest mass (why?). The decay schemes are shown in Figure

$$\tau^{-1} = \frac{G^2 m_\mu^5}{192\pi^3}$$

(1).

The experimental arrangement is illustrated in Figure

4. According to the range-energy relation for muons (see Rossi 1952, p40), a muon that comes to rest in 10 cm of plastic scintillator ($[CH_2]_n$ with a density of ≈ 1.2 g cm^3) loses about 50 MeV along its path. The average energy deposited by the muon-decay electrons in the plastic is about 20 MeV. We want both START and STOP pulses for the TAC to be triggered by scintillation pulses large enough to be good candidates for muon-stopping and muon-decay events, and well above the flood of < 1 MeV events caused mostly by gamma rays and the "after" pulses that often occur in a photomultiplier after a

strong pulse. The e□ect of "after-pulses" from the phototubes is eliminated by the use of two PM's and the coincidence requirement.

The success of the measurement depends critically on a proper choice of the discrimination levels set by the combination of the

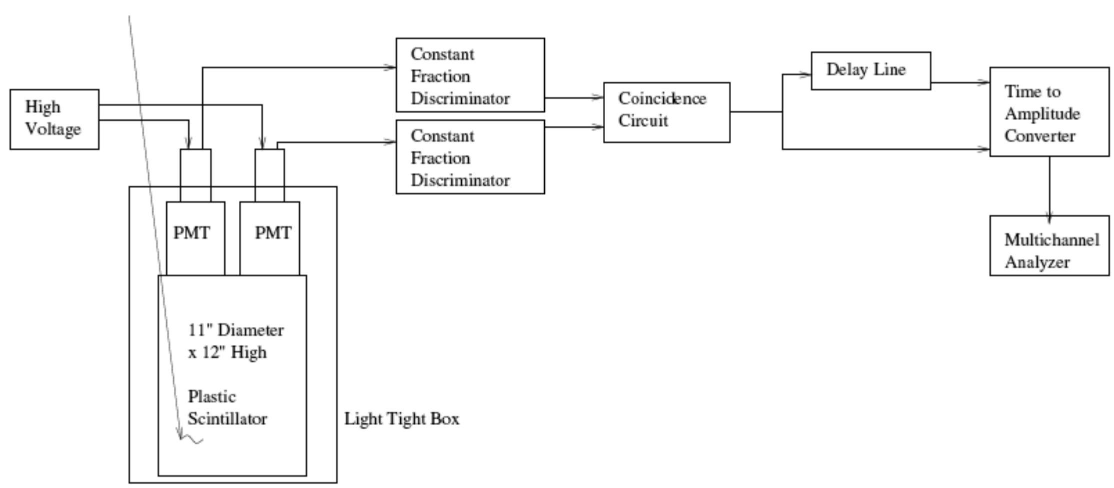
HV and the CFD settings. If they are too low, and the rate of accidental coincidences into the TAC is correspondingly too high, then the relatively rare muon decay events will be lost in a swamp of accidental delayed coincidences between random pulses. If the discrimination levels are too high, you will miss most of the real muon decay events. To arrive at a decision, review your prediction of the rate of decay events in the plastic cylinder. The answer to Preparatory Problem 5 tells you (implicitly) how to estimate the rate of accidental delayed coincidence events in which a random start pulse is followed by a random stop pulse within a time interval equal to, say, five muon mean lives. You want this rate of accidental events to be small compared to the rate of muon stoppings, allowing for reasonable inefficiency in the detection of the muon decay events due to the variability of the conditions under which the muons stop and the decay electrons are ejected.

To avoid inhibiting the timing sequences by the simultaneous arrival of every pulse at the START and STOP inputs of the TAC the pulses to the START input must be delayed with a sufficient length of coaxial cable to insure that their effect at the STOP input is finished before the timing sequence is initiated. Every pulse that triggers the discriminator should start a timing sequence which will be stopped by the next pulse that arrives at the STOP input, provided it occurs before the end of the TAC timing ramp. What effect does this necessary delay of the start pulse and the consequent loss of short-lived events have on the mean life measurement? A potential complication in this measurement is the fact that roughly half of the stopped muons are negative and therefore subject to capture in tightly bound orbits in the atoms of the scintillator. If the atom is carbon then the probability density inside the nucleus of a muon in a 1s state is sufficiently high that nuclear absorption can occur by the process (see Rossi, “High Energy Particles”, p 186)

$\mu^- + p \rightarrow n + v$, (12)

which competes with decay in destroying the muon. (Note the analogy with K-electron capture which can compete with positron emission in the radioactive decay of certain nuclei. Here, however, it is the radioactive decay of the muon with which the muon capture process competes.) The apparent mean life of the negative stopped muons is therefore shorter than that of the positive muons. Consequently the distribution in duration of the decay times of the combined sample of positive and negative muons is, in principle, the sum of two exponentials. Fortunately, the nuclear absorption rate in carbon is low so that its effect on the

Setup



PROCEDURE

FIG. 4: Arrangement for measuring the mean life of muons

Examine the outputs of the high gain photomultipliers with the oscilloscope. Adjust the high voltage supplies so that negative pulses with amplitudes of 1 volt or larger occur at a rate of the order expected for muon traversals (use Equation 3 to check this). Do not exceed 1850 V to keep the noise tolerable. Feed the pulses to the coincidence circuit. Examine the output of the coincidence circuit on the oscilloscope with the sweep speed set at 1 μsec cm^{-1}, and be patient. You should occasionally see a decay pulse occurring somewhere in the range from 0

to4or so μsec, and squeezed into a vertical line by the slow sweep speed. Now feed the negative output of the coincidence circuit directly to the STOP input and through an appropriate length of cable (to achieve the necessary delay as explained above) to the START input of the TAC. A suitable range setting of the TAC is 20.0 μsec, obtained with the range control on 0.2 μsec and the multiplier control on 100. Connect the TAC output to the MCA. Verify that most of the events are piling up on the left side of the display within a timing interval of a few muon lifetimes. Let some events accumulate and check that the median lifetime of the accumulated events is reasonably close to the half-life of muon. Calibrate the setup with the time calibrator.
Commence your measurement of muon decays. To record a su□cient number of events for good statistical accuracy you may have to run overnight or over a weekend. Plan your run so as to conform with the following schedule:

Schedule

Results aimed for

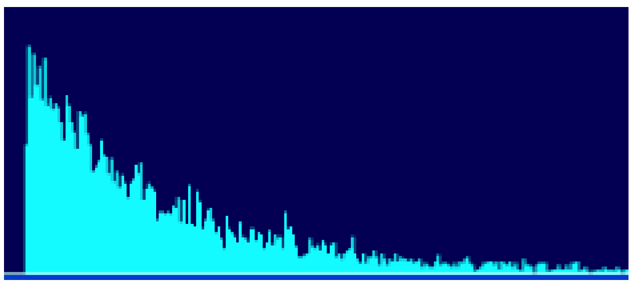


FIG. 7: Typical appearance on the MCA of the distribution in time of muon decays after about 10 hours of integration.

Guide to analysis

There is a potential pitfall in the analysis. The distribution in duration of intervals between successive random pulses is itself an exponential function of the duration, with a characteristic “decay” time equal to the reciprocal of the mean rate. If this characteristic time is not much larger than the muon lifetime, then the muon decay curve will be distorted and a simple analysis will give a wrong result. If the average time between events is much larger than the mean decay time, then you may assume that the probability of occurrence of such events is constant over the short intervals measured in this experiment, provided the triggering level is independent of the time since the last pulse. Under this condition, the observed distribution is a sum of a constant plus an exponential function of the time interval between the start and stop pulse. The constant, which is proportional to the rate of background events, is the asymptotic value of the observed distribution for large values of t. If this constant is subtracted from the distribution readout of the MCA, then the remainder should fit a simple exponential function the logarithmic derivative of which is the reciprocal of the mean life.

You can derive a value of the muon mean life by first determining the background rate from the data at large times, and then fitting a straight line by eye to a hand drawn plot of the natural logarithms of the numbers of counts minus background in successive equal time bins versus the mean decay time in the interval. You should also use a non-linear fitting algorithm to fitting the 3parameter function $n_i = ae$(18) to your data by adjusting a, b, and τ by the method of least squares, i.e. by minimizing the quantity $\chi^2 = (n_i - m_i)^2/m_i$, (19) where m_i is the observed number of events in the i th time interval. (Watch out for faulty data in the first few tenths of a μsec due to resolution smearing after pulsing of the photomultiplier, and the decay of negative muons that su□er loss by nuclear absorption.)

Consult Melissinos (1966) for advice on error estimation. Finally compare your fit value for b to the expected number of ‘acciden-tals’.

Calculate a typical value of the Lorentz factor γ at production of a muon that makes it to sea level and into the plastic scintillator.

1 What would be the vertical intensity of muons at an altitude of 10 km given their observed intensity at sea level if all cosmic ray muons were produced at altitudes above 10 km and time dilation were not true. How does this value compare with the actual value measured in balloon experiments? (See Appendix B for data on the flux versus atmospheric depth.)

Your paper

Evaluate 1. How long does it take a typical high energy cosmic-ray muon to get to sea level from its point of production? What would its survival probability be if its life expectancy were the same as that of a muon at rest?

References

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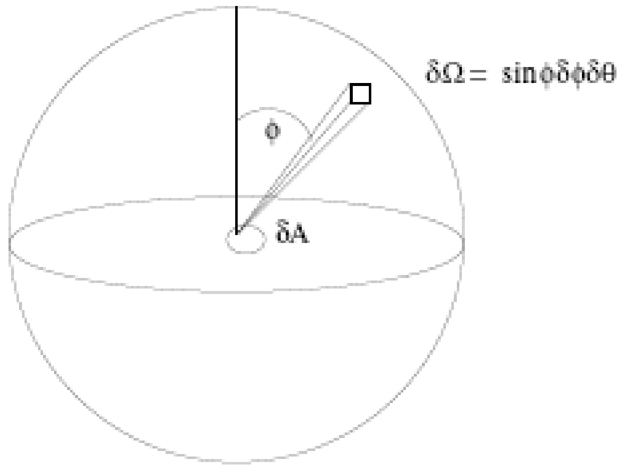
Appendices

APPENDIX A: PROPERTIES OF THE FLUX OF COSMIC-RAY MUONS

The di□erential flux I_v = dN(dAdtdΩ) is given in Fig. 9 for vertical (φ=0) incidences as a function of atmospheric depth. Sea level is ≈1000 g cm^{-2} areal density. (e.g. air, scintillators, etc.)
F



S = mI(Rav, 0). (A3)

FIG. 8: Di□erential element of the flux of cosmic-ray muons.

The distributions at other zenith angles can be represented fairly well by the empirical formula I(R, □)= I(R, 0) $\cos^2$ □. The stopping material in the experiment is a cylinder of scintillator plastic. Call its height b, its top area A , and its density ρ. Consider an infinitesimal plug of area dA in an infinitesimally thin horizontal slice of thickness (measured in g cm^{-2}) dR = ρdx of the cylinder. The stopping rate of muons arriving from zenith angles near □ in d□ in the element of solid angle dΩ in that small volume dA dx can be expressed as

ds = I(R□ , 0) $\cos^2$ □(cos □dA)(ρdx/ cos □)dΩ (A1)

where (cos □ dA) is the projected area of the plug in the direction of arrival, (dx/ cos □) is the slant thickness of the plug, and R□ is the residual range of muons that arrive from the vertical direction with just su□cient energy to penetrate through the overlying plastic to the elemental volume under consideration. The total rate S of muon stoppings in the cylinder can now be expressed as the

multiple
integral

in which we have replaced dΩ by 2πsin□d□ under the assumption of azimuthal symmetry of the muon intensity. Looking at Figure 11, we see that the muon range spectrum is nearly constant out to energies much greater than necessary to penetrate the building and the plastic. So we can approximate the quantity I(R□ , 0) by the constant I(R, 0). Performing the integrations and calling m = Abρ the mass of the entire cylinder, one readily finds for the total rate of muons stopping in the cylinder the expression

2π

4.1. Why muon decay is so very interesting

We now know that there are two oppositely charged muons and that they decay according to the following three body decay schemes:

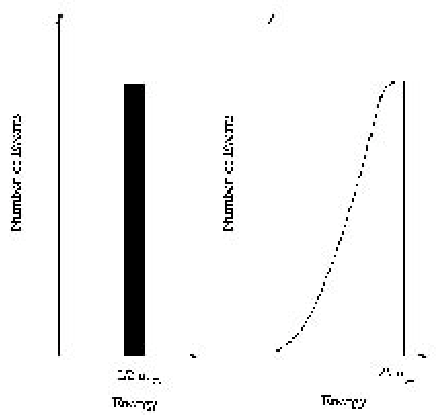
$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ (13a)

$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$ (13b)

Rossi's particle was falsely believed to be the one demanded by Yukawa, which in 1947 was found to be the pion at 140 MeV. However, the charged pion decays[1] into muons
two-body decay! We learned from this the following

The existence of a new kind of neutrino ν:

The energies of the decay electron in the pion and muon decay schemes look very different



Typical energy spectra resulting from two (left figure) and three (right figure) body decays.

The muons from pion decay are3rd body must have a spin=1/2. The 1988 Nobel
polarized anti-parallel to the flight direction and retain their polarization when stopping. The num

The decay $\pi \rightarrow e^- + \bar{\nu}_e$ is of course also possible but is suppressed by spin helicity. [2]
http://nobelprize.org/physics/laureates/1988/index.html

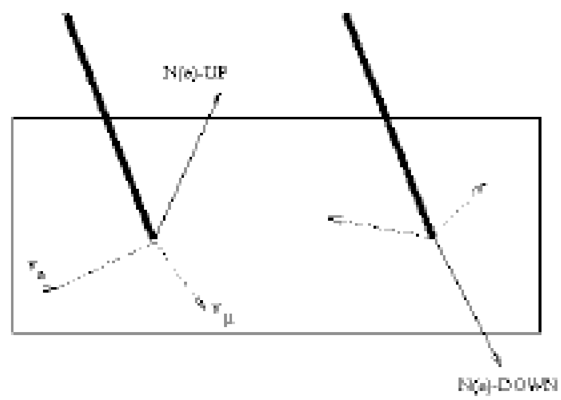


FIG. 6: Schematic of polarized muon decay demonstrating parity violation, i.e. $N(e)_{UP} \neq N(e)_{DOWN}$

APPENDIX B: DISTRIBUTION OF DECAY TIMES

The fundamental law of radioactive decay is that an unstable particle of a given kind that exists at time t will decay during the subsequent infinitesimal interval dt with a probability rdt, where r is a constant characteristic of the kind of the particle and independent of its age. Call P (t) the probability that a given particle that exists at t = 0 will survive till t. Then the probability that the particle will survive till t + dt is given by the rule for compounding probabilities,

$P (t + dt)= P (t)[1 - rdt]$. (B1)

$dP = -P rdt$, (B2)

from which it follows that $P (t)= e$. (B3)

To find the di□erential distribution of decay times n(t), which is the distribution measured in the muon decay experiment with the TAC and MCA, we multiply the negative derivative of P by the product of the rate S at which muons stop in the scintillator by the total time T of the run. Thus

$n(t)=(ST)(-dP/dt)=(ST)re$. (B4)

Identical reasoning can be applied to the problem of finding the distribution in duration of the intervals between random events that occur at a constant average rate s, like the background events in the muon decay experiment. In this case each random event that starts a timing operation, in e□ect, creates an 'unstable' interval (=particle) that terminates (=decays) at the rate s. Thus the distribution is a function of exactly the same form, namely

$m(t)=(sT)se$, (B5)

where (sT) is the expected total number of events in the time T . Note that the number of background events is proportional to s. This suggests a limit on how low the discriminator can be set in an e□ort to catch all of the muon stopping events. At some point the ratio of muon decay events to background events will begin to decrease as s.

From B. Rossi, Rev. Mod. Phys., **20**, 537 (194

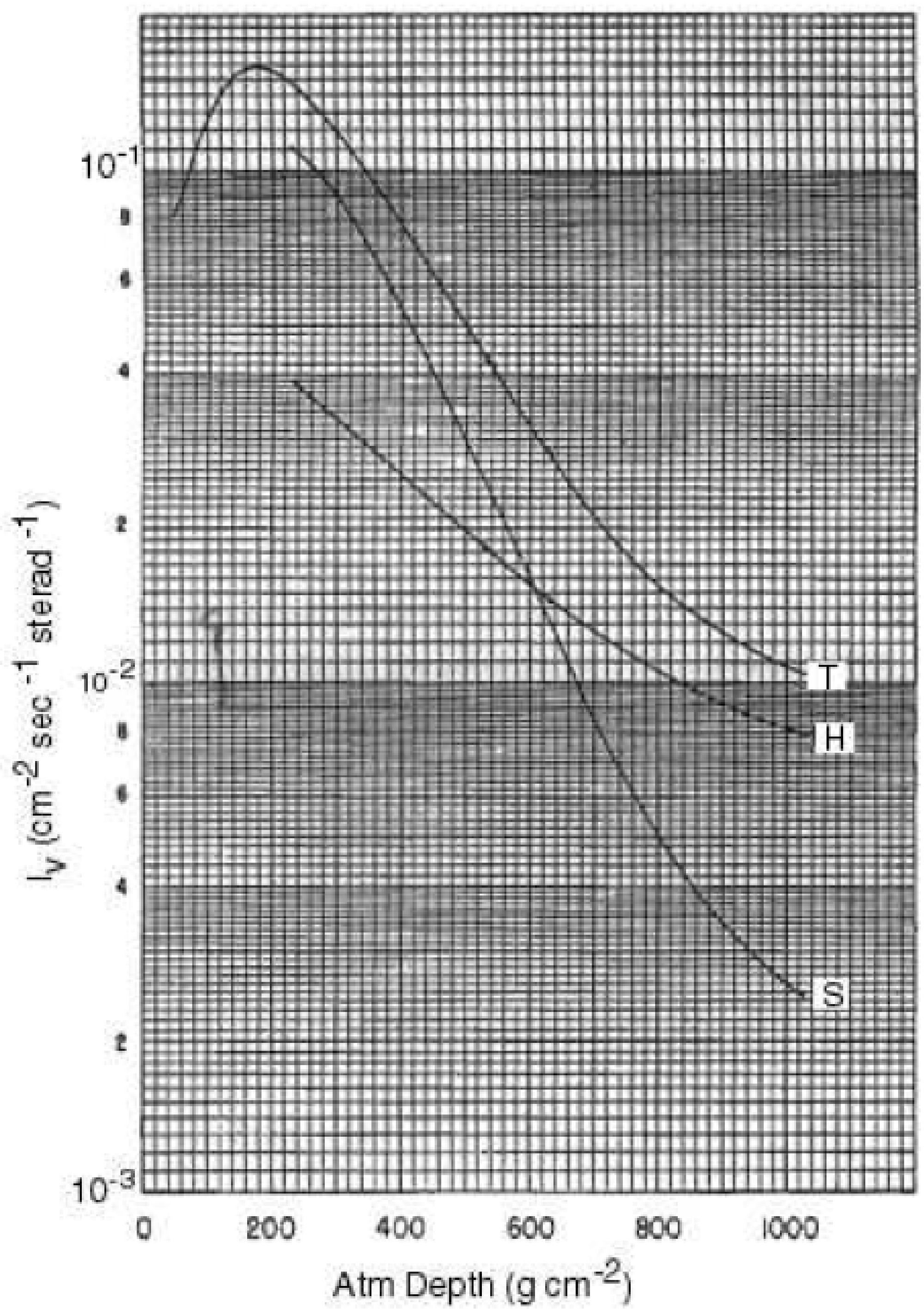


FIG. 9: The vertical intensities of the hard component (H), of the soft component (S), and of the total corpuscular radiation as a function of atmospheric depth near the geomagnetic equator.

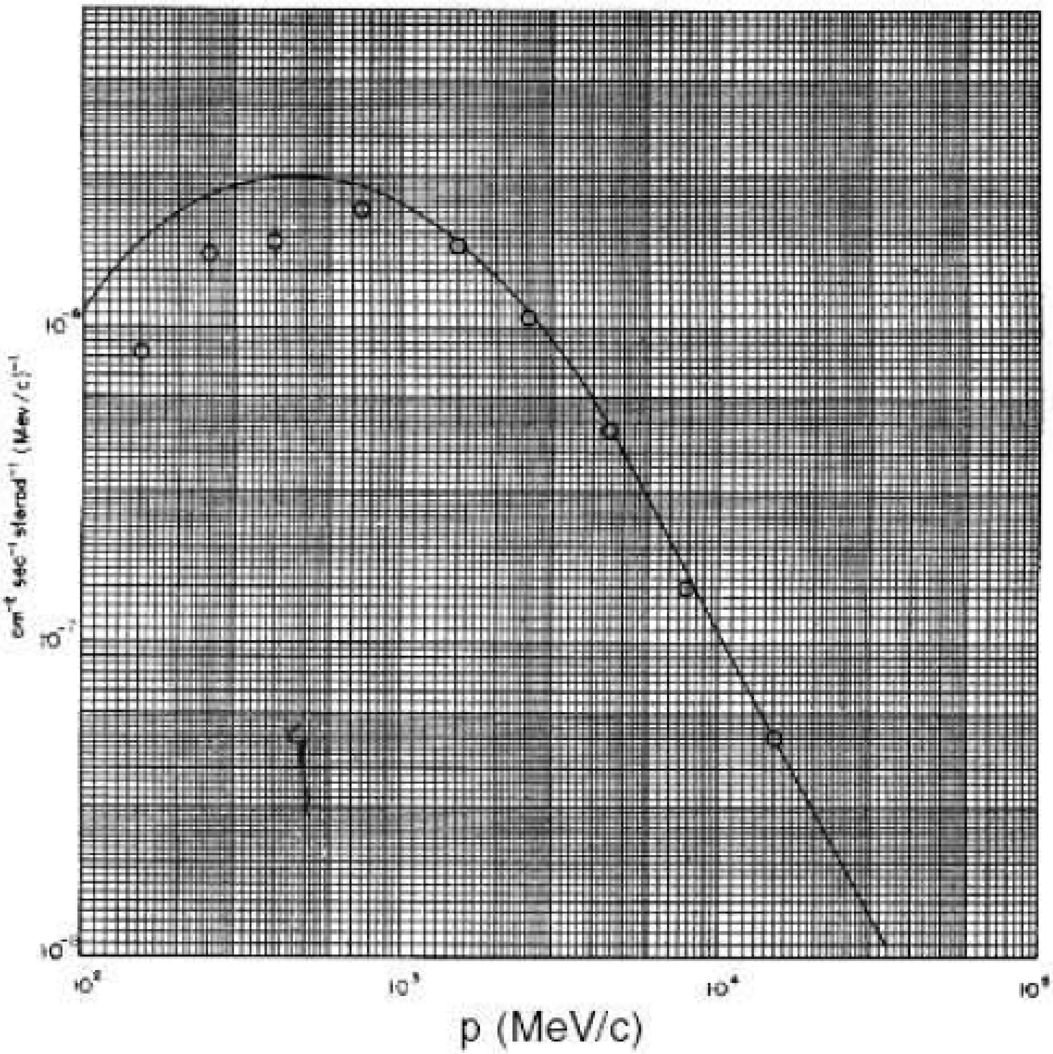


FIG. 10: Differential momentum spectrum of muons at sea level. The x-axis ranges from 10^2 to 10^5

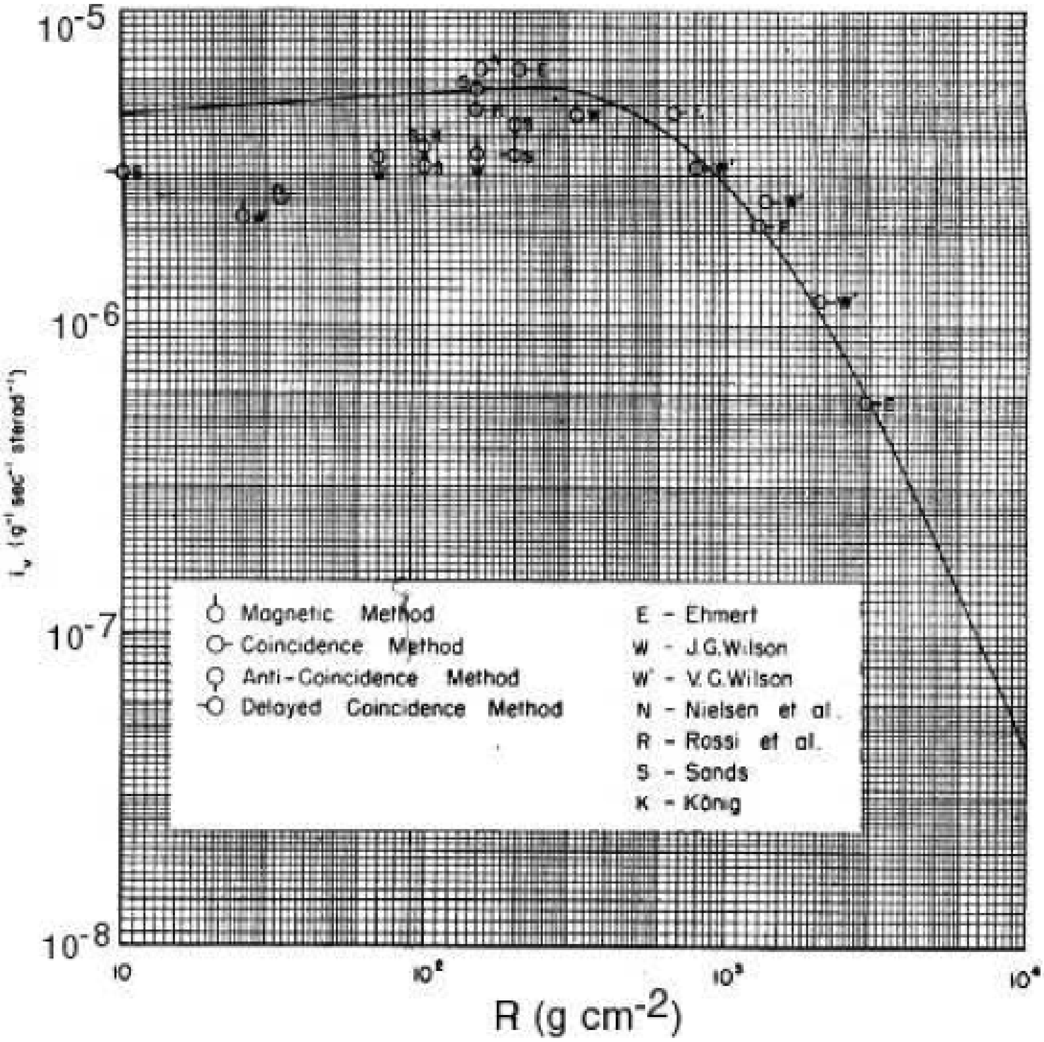


FIG. 11: Differential range spectrum of muons at sea level. The range is measured in gm cm^{-2} of air. The x-axis ranges from 10 to 10,000